

General Algebra

- Order of Operations: PEMDAS

Solve for 2 variables

- Substitution: 1) Solve 1 equation for 1 variable, 2) substitute that variable into the other equation, 3) solve
- Addition/Subtraction: If the coefficients of x or y are already opposite numbers, add the 2 equations to eliminate x or y

Nested Functions: start inside and work outwards

- $f(g(x))$ [standard notation]
- $(f \circ g)(x) = f(g(x))$
- $(f \cdot g)(x) = (fg)(x) = f(x) \cdot g(x)$

Absolute Values

- $|x| = x$ if $x \geq 0$, $|x| = -x$ if $x < 0$ & The result of an abs value > 0
- Use PEMDAS (absolute values comes after parentheses)

Inequalities

- Change inequality sign if the equation is divided by a negative

Equation of a Line and Midpoint, Distance

Slope: $m = \text{rise} / \text{run} = (y_2 - y_1) / (x_2 - x_1)$

Equations that describe a line

- Slope intercept form: $y = mx + b$
- Point-Slope Form: $y - y_1 = m(x - x_1)$

Parallel lines: $m_1 = m_2$

Perpendicular lines: $m_1 = -1 / m_2$

Midpoint Equation: $((x_1 + x_2) / 2, (y_1 + y_2) / 2)$

Distance Equation: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Quadratic Equation

- Polynomial equation: $ax^2 + bx + c = 0$
- Factored form: $(x + a)(x + b) = 0$ (use FOIL to go to the polynomial equation)
- Factors: $(x + a)$ & $(x + b)$
- Solutions, Roots, Zeros: $x = -a$, $x = -b$
- Zeros: the value of the factor when it crosses the x axis

Number Types

- Imaginary numbers: i is an imaginary number (Ex: $5i$, $2i$, $-3i$, i)
- Complex numbers: $a + bi$ (a is the real number, bi is the imaginary number)
- Values of imaginary numbers: $i = \sqrt{-1}$, $i^2 = -1$, $i^3 = -\sqrt{-1}$, $i^4 = 1$
- Integers: 3, 2, 9, 0
- Rational Numbers: Finite numbers (3, -2, 0, 2.25, 9.125)
- Irrational Numbers: 2.22333... π , $\sqrt{3}$

Matrices

- Notation: height x width (4 x 3 matrix means 4 rows & 3 columns)
- The matrices must be the same size to add or subtract
- The inner 2 size dimensions must be same to multiply matrices (4x3 & 3x2)

scalar multiplication $n \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} na & nb & nc \\ nd & ne & nf \end{bmatrix}$

matrix addition $\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} + \begin{bmatrix} g & h \\ i & j \\ k & l \end{bmatrix} = \begin{bmatrix} a+g & b+h \\ c+i & d+j \\ e+k & f+l \end{bmatrix}$

matrix multiplication $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} g & h \\ i & j \\ k & l \end{bmatrix} = \begin{bmatrix} ag+bi+ck & ah+bj+cl \\ dg+ei+fk & dh+ej+fl \end{bmatrix}$

Statistics

- Mean: average
- Median: middle number in a data set (or the average of the middle 2 numbers if there is an even set of numbers)
- Mode: most frequently occurring number(s)
- Range: Max value - Min value

LCM, LCD, GCF

- **LCM** (least common multiple): smallest multiple amongst several numbers or variables
- **LCD** (least common denominator): smallest common denominator for several fractions. It is used to add/subtract fractions
- **Divisible by**: a number can be divided into another number. Use the same method as LCM to determine the numbers
- **GCF** (greatest common factor): largest possible integer that divides into other numbers

Fractions

- Fraction Notation: Numerator / Denominator
- Add/Subtract fractions: denominators must be the same ($1/5 + 2/5 = 3/5$)...use the LCD

%

- A % can be written as a decimal or a % ($0.5 = 50\%$)
- Given a Tax or Tip %, the new amount = Original $(1 + \%)$
- Given a Discount %, the new amount = Original $(1 - \%)$
- % increase or decrease = $(\text{New} - \text{original}) / \text{original} * 100\%$

Proportions and Ratios

- Cross multiply: $a/b = c/d \Rightarrow ad = bc$
- Fractions or proportions and ratios are represented differently, but mean the same thing ($5 : 2 = 5/2$)

3 ways to represent a proportion or ratio

(All examples below: There is a ratio of 5 apples : 2 oranges)

- **Standard ratio**: In another batch, how many apples are there if there are 4 oranges $\Rightarrow 5 / 2 = x / 4$ & $5 : 2 = x : 4$
- **Ratio with a total**: If the total cost for all apples and oranges is \$10, how much is it for apples? $\Rightarrow 5$ apples / 7 fruit = \$x apples / \$10 total & $5 : 7 = x : 10$
- **2+ variable equation**: If 1) the number of apples = the number of oranges & 2) the total cost for all apples & oranges = \$10, how much is it for apples? $\Rightarrow 5x + 2x = \$10$

Graphing

- Undefined values in the domain are found by setting factors in the denominator = 0. Undefined values show up as discontinuities and asymptotes on a graph
- 1) Discontinuity: a break in a line that shows up as an open circle in a graph
- 2) Undefined values & vertical asymptotes are all other undefined values in the domain. They can also be values when a square root is < 0

Exponents

- $(a^x)^y = a^{xy}$
- $(ab)^x = a^x \cdot b^x$
- $a^x \cdot a^y = a^{x+y}$
- $(a/b)^x = a^x / b^x$
- $a^{-x} = 1 / a^x$
- $a^0 = 1$

Radicals

- Radicals or Roots: $\sqrt[n]{x}$ reads, x radical n or the nth root of x. It can also be written $x^{1/n}$

Logarithms

Log Notation: $\log_b b^r = \log_{10} b^r$
Natural Log Notation: $\ln_e x$

Logarithm Properties

- $\log_a b^c = c \log_a b$
- $\log_a a^r = r$
- $\log_a b^{-1} = -\log_a b$
- $\log_a (b * c) = \log_a b + \log_a c$
- $\log_a (b / c) = \log_a b - \log_a c$

- log of 0 and negative numbers is undefined

Scientific Notation

- $a \times 10^{\text{exponent}}$
- a: place decimal after the 1st digit (3.28)
- Add / Subtract terms: add bases when exponents are the same
- Multiply / Divide: multiply a terms and exponent terms separately







Sequences & Patterns

- Arithmetic Sequence: series of numbers with a constant difference (d)
- $d = a_n - a_{n-1}$
- $a_n = a_1 + (n - 1) \cdot d$
- Geometric Sequence: series of numbers that change by the same rate (r)
- $r = a_n / a_{n-1}$
- $a_n = a_1 \cdot r^{n-1}$

Similar Triangles & Triangle Types

- Corresponding angles are equal (or congruent)
- Corresponding sides are in the same proportion, $a/p = b/q = c/r$

Triangle Types

| By Side | By Angle |
|---|--|
|  Equilateral Triangle has three equal sides |  Acute triangle has three angles $< 90^\circ$ |
|  Isosceles Triangle has two equal sides |  Right triangle has one angle $= 90^\circ$ |
|  Scalene Triangle has no equal sides |  Obtuse triangle has one angle $> 90^\circ$ |

Angles

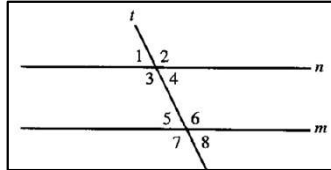
Sum of Angles

- 3-sided figure (triangle) = 180°
- 4-sided figure (square, rectangle) = 360°
- 5-sided figure (pentagon) = 540°
- $S = 180^\circ * (n-2)$ (n = number of sides, S = angle)

- Bisector - divides 1 angle into 2 equal angles

Vertex angles and Parallel Lines

- Opposite angles are = (ex: $\angle 1 = \angle 4$)
- Opposite interior angles = (ex: $\angle 3 = \angle 6$)
- Corresponding angles = (ex: $\angle 1 = \angle 5$)



- Radians = $180^\circ / \pi * \text{Degrees}$

- Central angle of a circle is proportional to the perimeter or area for a sector (ex: Central angle $(\theta) = 360^\circ * \text{circumference of sector} / \text{circumference of circle}$)

Trigonometry

Use only with right triangles

sohcahtoa

- $\cos \theta = \text{adj} / \text{hyp} = b/c$
- $\sin \theta = \text{opp} / \text{hyp} = a/c$
- $\tan \theta = \text{opp} / \text{adj} = a/b$

- $\cos^{-1}(b/c) = \arcsin(b/c) = \theta$
- $\sin^{-1}(a/c) = \arcsin(a/c) = \theta$
- $\tan^{-1}(b/a) = \arctan(b/a) = \theta$
- Angle of inclination / ascent / elevation: angle between horizontal line & the slanted line or hypotenuse

- Pythagorean Theorem: $a^2 + b^2 = c^2$

Perimeter, Area, Volume

Circles

- Circumference = $2 \pi r$
- Area = πr^2
- $r = 1/2 * d$

Spheres

- Volume = $4 \pi r^3 / 3$

Triangles

- Perimeter = leg 1 + leg 2 + leg 3
- Area = $1/2 b * h$
- Volume = $4 \pi r^3 / 3$

Squares & Rectangles

- Perimeter = $2 * l + 2 * w$
- Area = $l * w$
- Surface Area = $6 * l * w$
- Volume = $l * w * h$

Mixed Shapes

1. Add shapes: Add areas or perimeters of different shapes
2. Shape inside another shape: Determine the area of the space inside the large shape, but not in the small shapes
3. Compare shapes: The sides / areas / perimeters are similar by a factor of x

Parallelograms

- Area = $b * h$
- Opposite sides and angles are equal

Trapezoids

- Area = $1/2 (b_1 + b_2) h$

Right Circular Cylinder

- $V = \pi r^2 h$

Right Circular Cone

- $V = 1/3 \pi r^2 h$ (formula given)

Rhombus

- $A = b h$ (b = base, h = height)
- $A = 1/2 * \text{diag 1} * \text{diag 2}$
- Opposite sides are = in length

Conics

- Circle: $(x - h)^2 + (y - k) = r^2$ (Center is (h,k), radius is r, (x,y) is any point on the circle)

Parabola equation: $y = k(x - a)^2 + b$

- $k > 0$ or $k < 0$ = opens up/down
- b = shift up & down & y coordinate of vertex

Probability

- Probability 1 Event Occurs = Number of desired events / Total number of possible events
- Look for the word, "or" for Probability 1 event occurs

- Probability > 1 event occurs = Probability Event A \cdot Probability Event B, etc
- Looks for the word, "and" for Probability > 1 event occurs

Combinations & Permutations

- Number of combinations = Number of Event A \cdot Number of Event B \cdot Number of Event C